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A SIMPLIFIED APPROACH FOR APPROXIMATE ANALYSIS AGAINST LATERAL LOADS

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ABSTRACT

A simplified approach that reduces the size of the problem to a more viable size is presented for estimating straining actions and drift values for preliminary design against lateral loads. The proposed technique does not dispense any of the main features that affect the structural behavior. The idea is an extension of the sub-frame method proposed by ACI for analysis against gravity loads. A one-story three-dimensional model of the actual building is proposed in order to provide estimates of forces, moments, and drift values for multi-story buildings. The results of a statistical study are used to develop analysis charts for framed systems with and without shear walls. Different cases of torsion eccentricities are considered. The study is conducted for up to 20-story buildings with different height-width ratios. Equations resulting from nonlinear regression analyses are presented in order to provide simplified expressions for preliminary analysis of multi-story buildings. Five cases are used to validate that the method provides good approximations that are adequate for design of multi-story buildings.

Keywords: structures; lateral; seismic; analysis; approximate; model.

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INTRODUCTION

For decades, it had been an engineering practice that engineers resort to various simplifications to make an unsolvable problem solvable or to cut short tedious and prolonged calculations. The principle was totally justifiable prior to the existence of computers. However, the trend is still carrying on even after the advent of high-speed number crunching machines. Inaccurate representation of loads, materials, geometrical properties or behavior, insufficient computational or financial resources, and avoidance of numerical drawbacks related to solution of large and complex problems, are some of the reasons for this trend. Inevitably, it seems that such approximations will exist as long as engineering exits. We limit our review, hereafter, to some of the famous approximations that take place in the area of structural analysis.

Design codes such as Egyptian Code (ECP'01 2001) and American Code (ACI’99 1999) allow approximate and simplified methods for determining analytical values used for the design of different members. For example, they allow approximate bending moment values for continuous slabs, beams, and columns as given in Secs. 6.2.2.4, 6.3.1.6, and 6.4.5.2 of ECP’01 and Secs. 8.3.3, 10.12, and 10.13 of ACI’ 99, respectively. Bowles (1982) allow similar approximations for mat foundations. Plane (2D) analysis of frames and related solution methods (Hibbeler 2002 and IDARC 2D 1996) - rather than space (3D) analysis had been an acceptable standard practice for decades. Existing linear first-order analysis program had been modified to produce acceptable results for nonlinear second order P–Δ effects (Nilson 1991 and Kamal 1994). In finite elements, a continuous problem described by a differential equation is represented by approximation functions satisfying certain boundary conditions at discrete points (Ainsworth 2000).

As far as lateral load analysis is concerned, it usually essential to reduce the size of the analysis problem by representing some of the structure’s assemblies by simpler analogous components. Smith and Coull (Smith 1991) discuss several different practical methods of analysis developed for the range of structural forms encountered in tall buildings. Mohammadi (2002) uses a statistical approach to develop an empirical formula for evaluating maximum inelastic deflection of multi-degree-of-freedom, *MDOF*, structures against seismic loads. Miranda (2000) conducts a comprehensive statistical study and nonlinear regression analyses to provide a simplified expression to be used in the design to approximate mean inelastic lateral displacements ratios for structures on firm sites. Miranda (2002) presents a method that provides good approximations for lateral drift demands in multi-story buildings with non-uniform stiffness that could be used for preliminary design of buildings.

In this work, a statistical study and nonlinear regression analyses are conducted to develop analysis charts and analysis equations in order to provide estimates for straining actions and drift values of multi-story buildings based on one-story three-dimensional structural model approach. To this end, the rest of this work is organized as follows. First, the proposed structural model is introduced. Next, structural layouts with different height-width ratios and different eccentricities are considered and analyzed. Finally, conclusions and discussions are presented.

THE PROPOSED MODEL

Usually a three-dimensional analysis of a fully detailed model of a structure presents a too formidable task of bookkeeping or computations. ACI'99 and earlier versions allow using a model limited to the beams in the level considered and the columns above and below that level, with far ends of columns considered as fixed for the purpose of analysis under gravity loads. This reduces the analysis process to a one-story-at-a-time procedure. The approach triggers the following question: Is it possible to use a one-story model to develop an approximate method of analysis against lateral forces?

In order to answer this question, an appropriate model is required. A structural model for an accurate analysis should represent in a detailed way all the major active components of the structure. The principal ones are the columns, shear walls, and connecting slabs and beams. In this work, a one-story three-dimensional model is proposed. This model is defined typically as the lower story of the multi-story building at hand. Slab, beam, column and shear wall sizes and locations are the same as those of the actual lower story of the building being considered. The total base-shear force V calculated using the equivalent static method (ECP'93 1993) for the multi-story building is applied at the slab level at the center of mass of the one-story model of the building, as shown in Fig.1. The results of the analysis of the actual building are compared with the proposed model. Certain correlation of straining actions and drift values between both cases is realized and translated into several curves and equations. In effect, it is proven possible to find estimates of the analysis values for a multi-story building through solving it's one-story model and scaling the results using the analysis aids developed this work.

Figure 2 shows the plan layout for the five cases considered in this work. All stories are three meters in height. All columns are spaced five meters in both directions. Framing action is provided by beams existing along all plan axes in both directions of each story. All column bases are assumed fixed. A horizontal diaphragm is considered at each story level in order to simulate the lateral drift of this story in the direction of the applied lateral load. Earthquake forces for all cases are considered in the direction shown in the figure. The buildings are assumed to be constructed in Cairo (zone 2) with an importance factor of *1* and a soil factor of *1.15*. Other factors are incorporated as per ECP'93.

For each case, several heights are considered. Buildings with two, five, ten, fifteen, and twenty stories with heights, h, ranging from 6 to 60ms are studied. For each height, different widths are used. Building widths, B, of 15, 20, 25, and 30ms are investigated. Note that all h/B values are within the limits allowed by ECP'93 for use with the equivalent static method. The six reaction values at each column base: shear in the x-direction (V_x), shear in the y-direction (V_y), torsion (M_t), normal forces (N), moment about x-axis (M_x), moment about y-axis (M_y), and the total drift of the building (Δ), are calculated for the full-scale building and for the corresponding one-story model shown in Fig. 1.

It is important to note that the ratios of the full-scale analysis to the one-story model analysis for matching columns for a specific straining action of significant design value are practically the same with very minor variations. Therefore, it is a good approximation to use an average ratio in this work. Also, it should be noted that the drift values considered here are those of the top story in the direction of the applied base-shear. A shear wall straining action in this work is represented by the resultant of the straining actions at all nodes of this wall. The relation between the height, h, of the building in meters (x-axis) and the ratio, R_i, of a specific

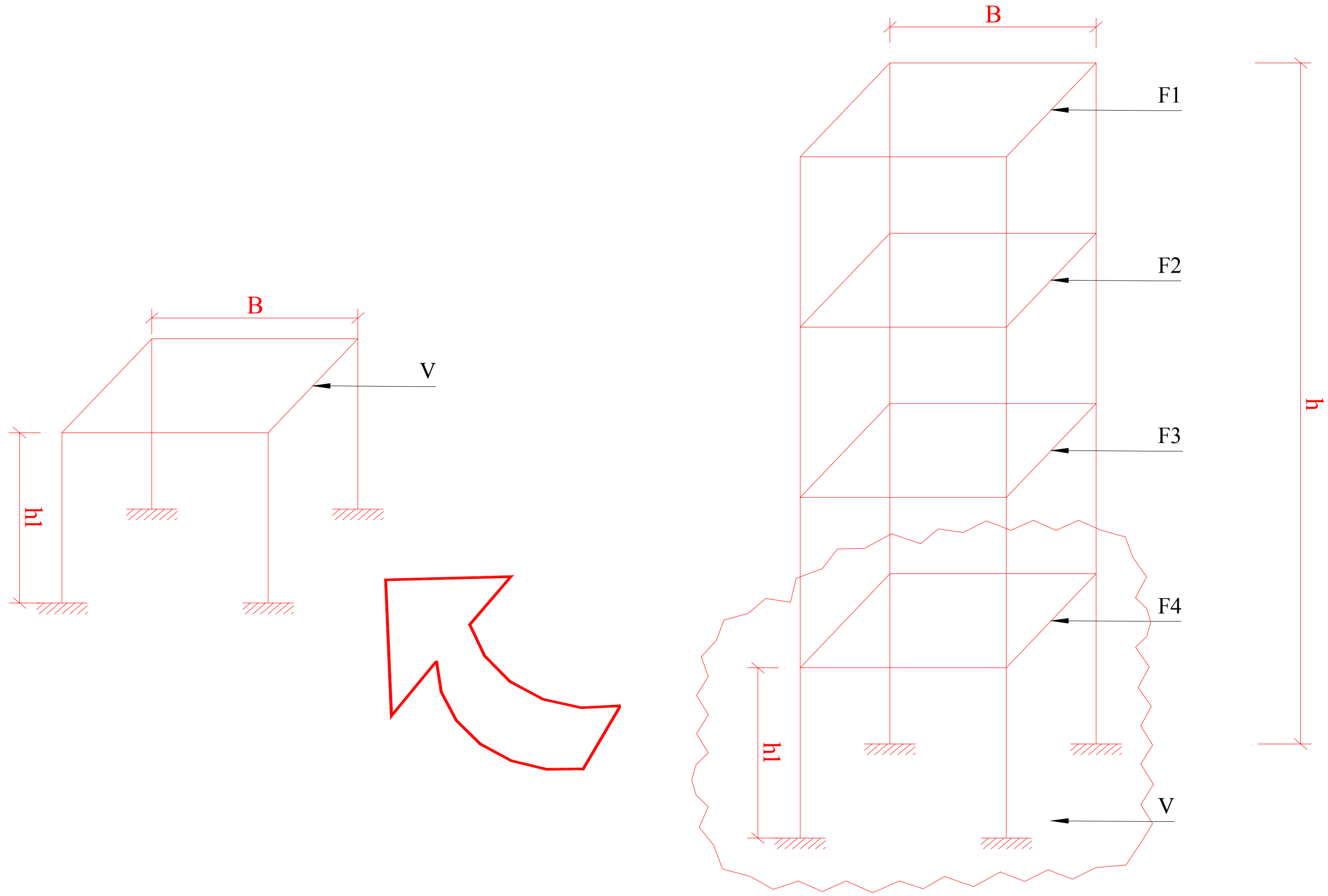

Fig.1. Proposed One-Story Three Dimensional Model

straining action or drift, i, of the full-scale structure to the proposed one-story model (y-axis) is recorded. A detailed description of each case is summarized hereafter.

Case (1): **Symmetric Moment-Resisting Frames.**

Figure 2 displays a symmetrical plan of a multi-story building, case (1), with a lateral load resisting system composed of moment-resisting frames. Considering the aforementioned combination of heights and widths, a total of 20 full-scale structure analyses and corresponding 20 one-story model analyses are investigated for this layout. Figures 3, 4, 5, and 6 show the drift Δ, shear force V_x, normal force N, and moment M_y ratios, respectively. Other straining action values: V_y, M_t, and M_x are negligible, and hence, not included in the figures. Values for $B=20$ and $B=25$ms are found by linear interpolation between the curves of $B=15$ and $B=30$ms. The curves obtained in these graphs suggest using nonlinear regression analysis to come up with mathematical formulae for the desired ratios. The following equations outline the governing relations for ratios of different straining actions and drift for this case:

(a) Lateral Drift, Δ (Fig. 3)

$$\left.\begin{aligned} \mathrm{B} = 15, f(h) &= 0.0026h^3 - 0.0556h^2 + 1.3028h - 4.3222 \\ \mathrm{B} = 30, f(h) &= 0.0021h^3 - 0.0425h^2 + 1.0979h - 3.4797 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (1)$$

(b) Horizontal Shear, V_x (Fig. 4)

$$\text{all B}, \quad f(h) = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

(c) Normal Force, N (Fig. 5)

$$\left.\begin{aligned} \mathrm{B} = 15, f(h) &= 0.0027h^3 - 0.0755h^2 + 1.0795h - 1.205 \\ \mathrm{B} = 30, f(h) &= 0.0025h^3 - 0.0874h^2 + 1.8597h - 6.7868 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (3)$$

(d) Moment M_y (Fig.6)

$$\left.\begin{aligned} \mathrm{B} = 15, f(h) &= 0.0007h^2 + 0.0155h + 0.9549 \\ \mathrm{B} = 30, f(h) &= 0.0006h^2 + 0.0136h + 0.9609 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The use of this approach is quite straightforward. For example, consider a multi-story building with a total height of 50ms. Using $h=50$ms, the corresponding R_i, multiplier value for straining action i is determined from Figs. 3-6 or equations (1)-(4). Next, apply the total V force obtained for the 50ms story building to the one story model of the building, as shown in Fig. 1. Find the one story building drift or straining action value and scale each item with its corresponding R_i multiplier to come up with the approximate values for the drift or straining action for the 50ms high building.

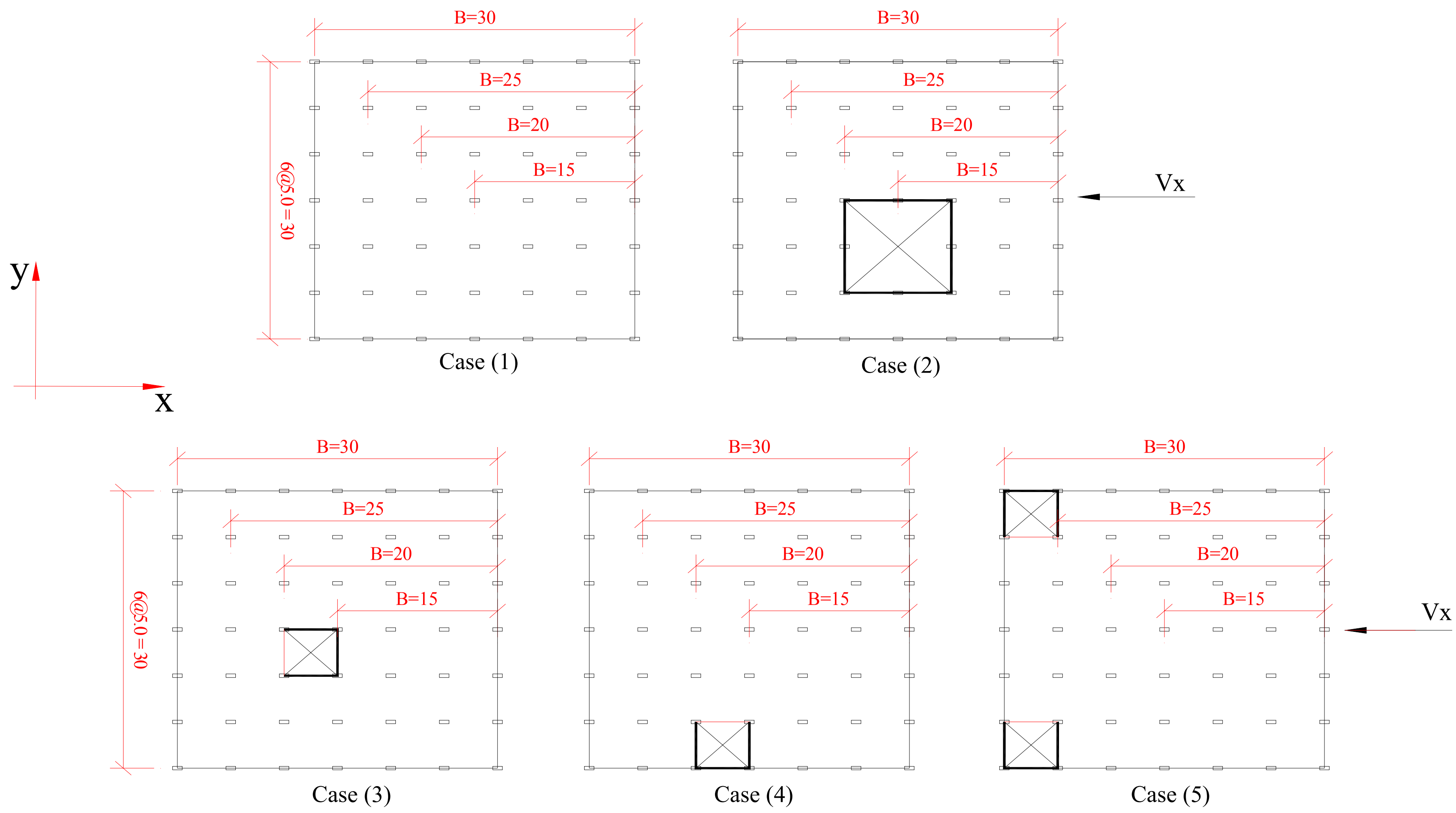

Fig.2. Plan Layouts for the Cases Investigated

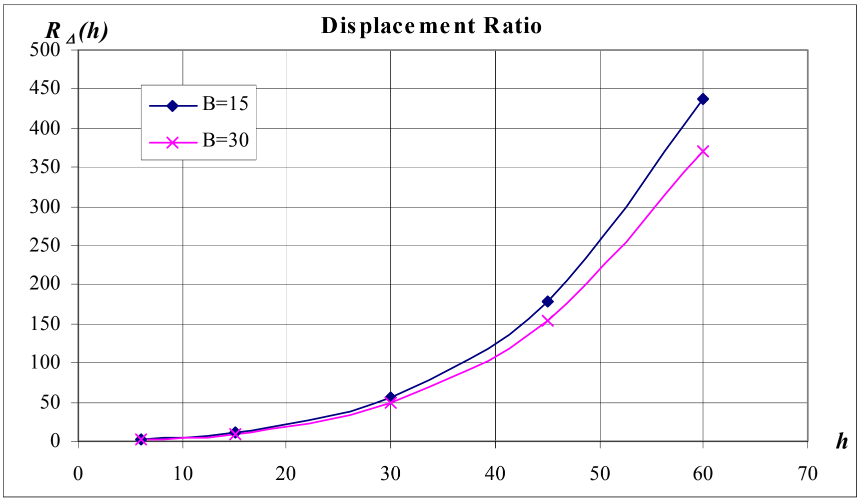

Fig 3. Case (1): Displacemet Ratio

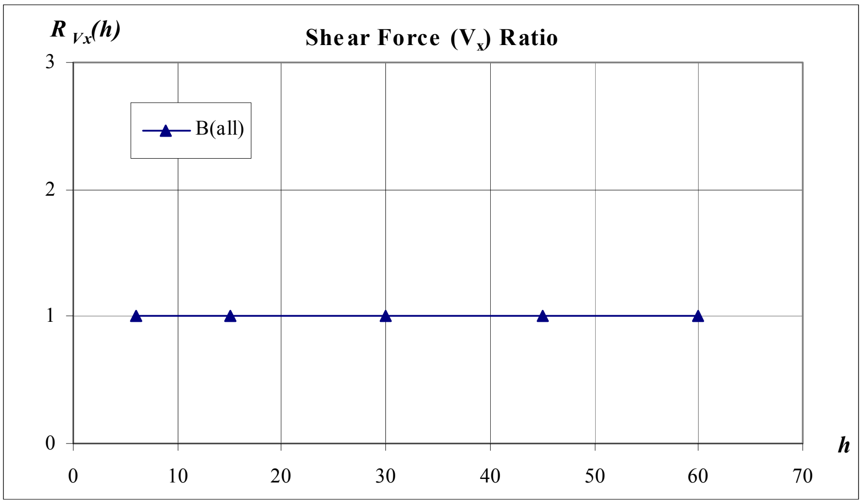

Fig 4. Case (1): Shear Force (Vx) Ratio

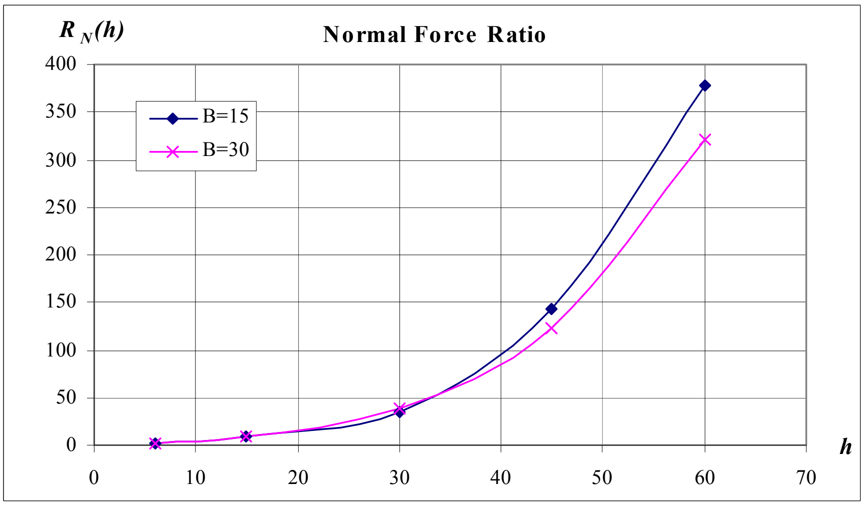

Fig 5. Case (1):Normal Force Ratio

Case (2): Eccentric Moment-Resisting Frames

Case (2) in Fig. 2 represents an unsymmetrical plan due to the presence of a *10x10*ms opening. The system is composed of ductile moment resisting frames. Several heights are considered: *6, 15, 30, 45*, and *60* meters. For each height, widths of *30, 25*, *20*, and *15* meters are investigated. The following relationships and figures govern the ratio of the full-scale building to the one-story model:

(a) Lateral Drift, $\varDelta$ (Fig. 7)

$$\left.\begin{aligned} B = 15, f(h) &= 0.0026h^3 - 0.054h^2 + 1.2762h - 4.2055 \\ B = 30, f(h) &= 0.0021h^3 - 0.0409h^2 + 1.0716h - 3.3635 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(5)$$

(b) Normal Force, N (Fig. 8)

$$\left.\begin{aligned} B = 15, f(h) &= 0.0029h^3 - 0.1053h^2 + 2.1886h - 8.2588 \\ B = 30, f(h) &= 0.0026h^3 - 0.0965h^2 + 2.0483h - 7.6503 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(6)$$

(c) Moment M_y (Fig. 9)

$$\left.\begin{aligned} B = 15, f(h) &= 0.0007h^2 + 0.0158h + 0.9553 \\ B = 30, f(h) &= 0.0006h^2 + 0.0146h + 0.9552 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(7)$$

Figures 7, 8, and 9 represent the ratios for the drift $\varDelta$, normal force N, and moment M_y, respectively. Figure 4 shows the ratio for the shear force V_x. Other straining actions: V_y, M_t, and M_x are negligible for all practical purposes.

Case (3): Dual Systems with Small Eccentricity

Figure 2 represents a layout of a lateral-load resisting system composed of a centric shear wall and moment-resisting frames, case (3). Several heights are considered: *6, 15, 30, 45*, and *60*ms. For each height, widths of *30, 25, 20*, and *15* meters are investigated. The following relationships and figures govern the ratio of the full-scale building to the one-story model for the columns and the shear wall:

(a) System Drift, $\varDelta$ (Fig. 10)

$$\left.\begin{aligned} B = 15, f(h) &= 0.0021h^3 - 0.029h^2 + 0.937h - 2.7503 \\ B = 30, f(h) &= 0.0018h^3 - 0.0258h^2 + 0.9169h - 2.6989 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(8)$$

(b) Column Normal Forces (Fig. 11)

$$\left.\begin{aligned} B = 15, f(h) &= 0.0025h^3 - 0.0878h^2 + 2.3253h - 9.1348 \\ B = 30, f(h) &= 0.0021h^3 - 0.0724h^2 + 1.824h - 6.3305 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(9)$$

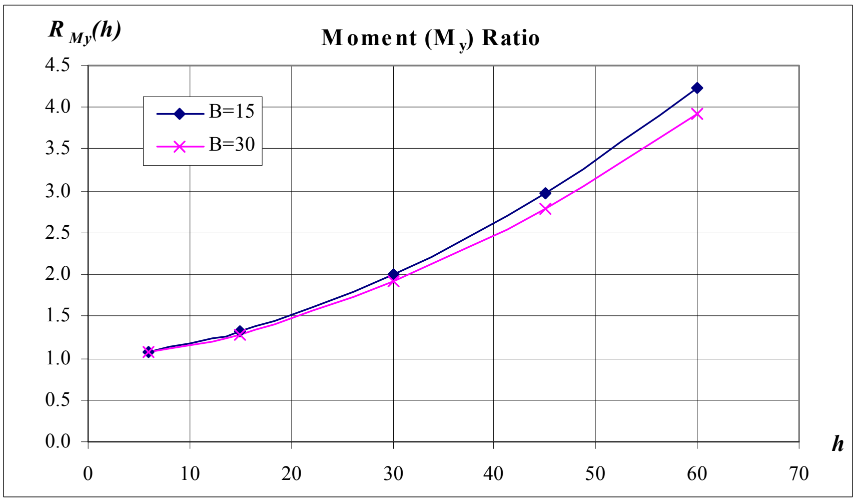

Fig 6. Case (1):Moment (My) Ratio

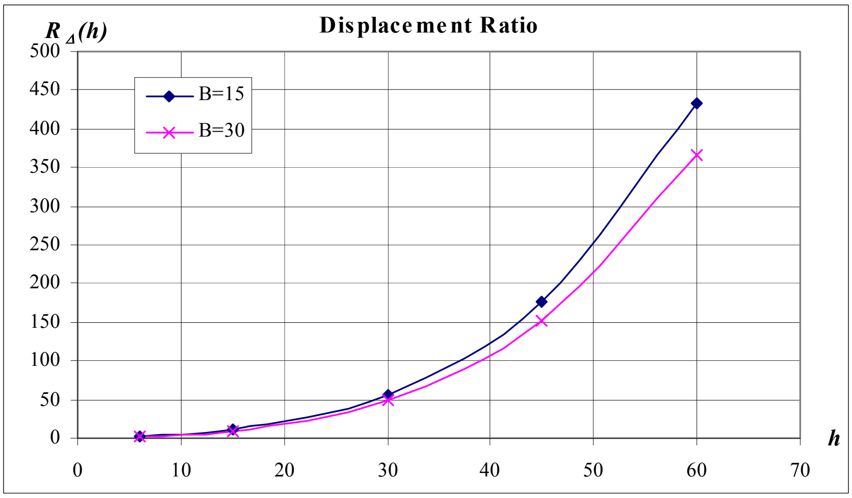

Fig 7. Case (2): Displacemet Ratio

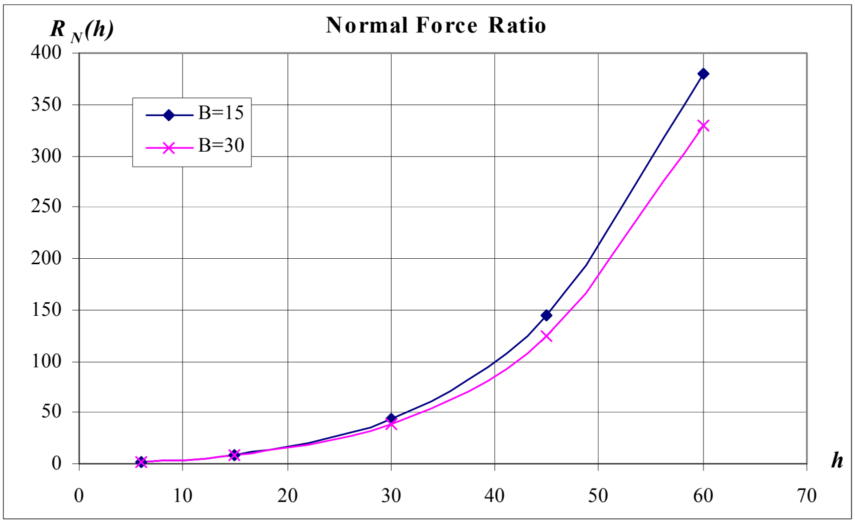

Fig 8. Case (2):Normal Force Ratio

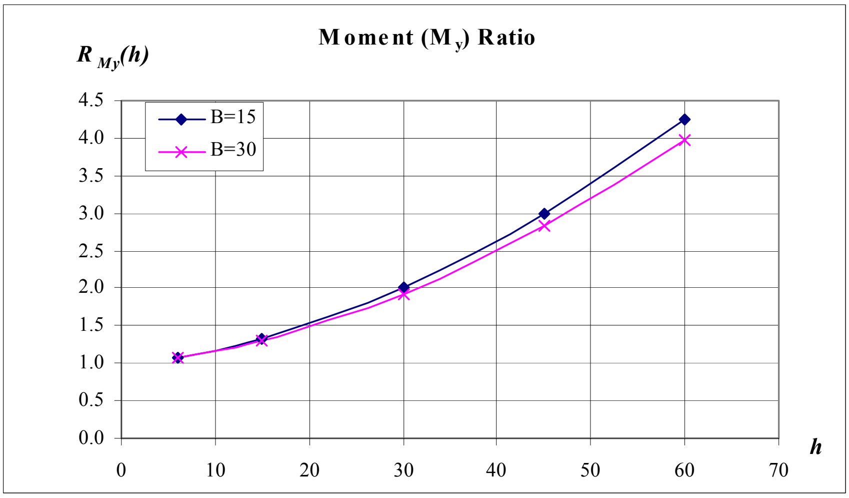

Fig 9. Case (2):Moment (My) Ratio

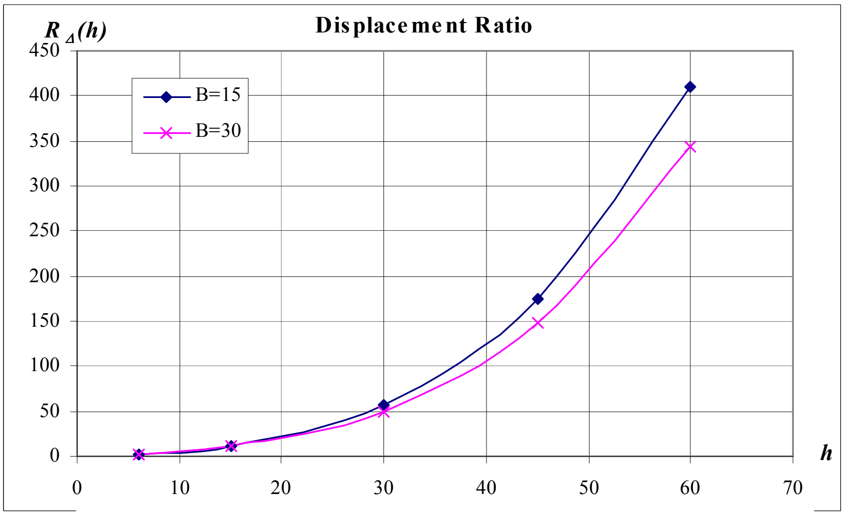

Fig 10. Case (3):Displacement Ratio

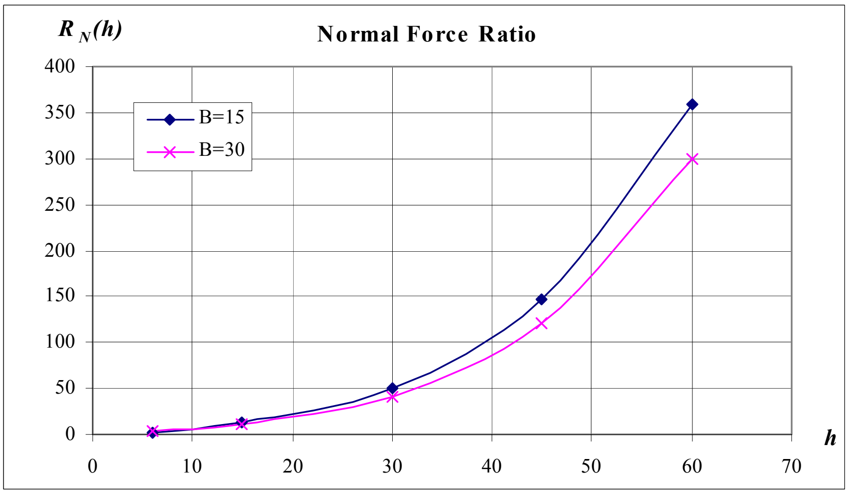

Fig 11. Case (3):Normal Force Ratio

(c) Column Moments M_y (Fig. 12)

all B, $f(h) = 0.0005h^2 + 0.0121h + 1.0174$ ……………………… (10)

(d) Shear Wall Moment M_y (Fig. 13)

……………………… (11)

all B, $f(h) = 0.0006h^2 + 0.11h + 1.0554$

Figures 10, 11, 12, and 13 represent the ratio for system drift Δ, column normal forces N, column moments M_y, and shear wall moment M_y, respectively. Figure 4 shows the ratio for the columns and shear wall shearing forces V_x. Other straining action values for columns and shear wall are negligible.

Case (4): Dual Systems with Big Eccentricity

Figure 2 represents a layout of a lateral-load resisting system composed of an eccentric shear wall and moment-resisting frames, case (4). Several heights are considered: *6, 15, 30, 45*, and *60*ms. For each height, widths of *30, 25, 20*, and *15* meters are investigated. The following relationships and graphs govern the ratio of the full-scale building to the one-story model for the columns and the shear wall:

(a) System Drift, Δ (Fig. 14)

$$\left.\begin{aligned} B = 15,\ & f(h) = 0.0026h^3 - 0.0568h^2 + 1.2418h - 3.6861 \\ B = 30,\ & f(h) = 0.002h^3 - 0.0386h^2 + 0.9773h - 2.6972 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (12)$$

(b) Columns Normal Forces (Fig. 15)

$$\left.\begin{aligned} B = 15,\ & f(h) = 0.0026h^3 - 0.0899h^2 + 1.8796h - 6.549 \\ B = 30,\ & f(h) = 0.0021h^3 - 0.0721h^2 + 1.5859h - 5.3587 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (13)$$

(c) Column Moments M_y (Fig. 16)

all B, $f(h) = 0.0005h^2 + 0.0116h + 0.9834$ ……………………… (14)

(d) Shear Wall Moment M_y (Fig. 17)

all B, $f(h) = 0.0016h^2 - 0.1707h + 0.682$ ……………………… (15)

Figures 14, 15, 16, and 17 represent the ratio for system drift drift Δ, column normal forces N, columns moments M_y, and shear wall moment M_y, respectively. Figure 4 shows the ratio for the columns and shear wall shearing forces V_x. Other straining actions for columns and shear wall are negligible.

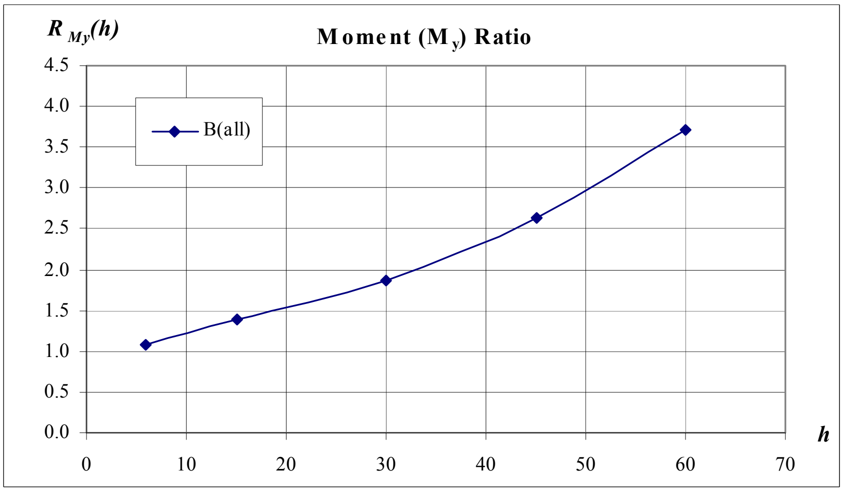

Fig 12. Case (3):Moment (My) Ratio

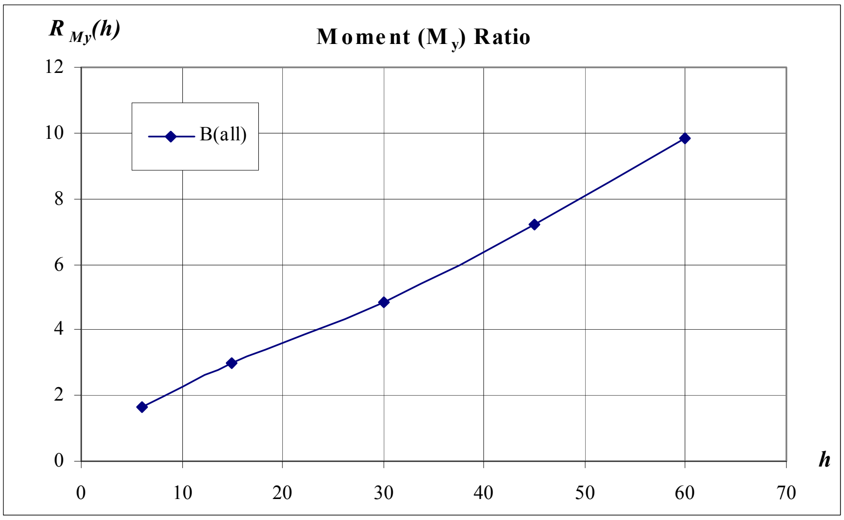

Fig 13. Case (3):Moment (My) Ratio - Shear Wall

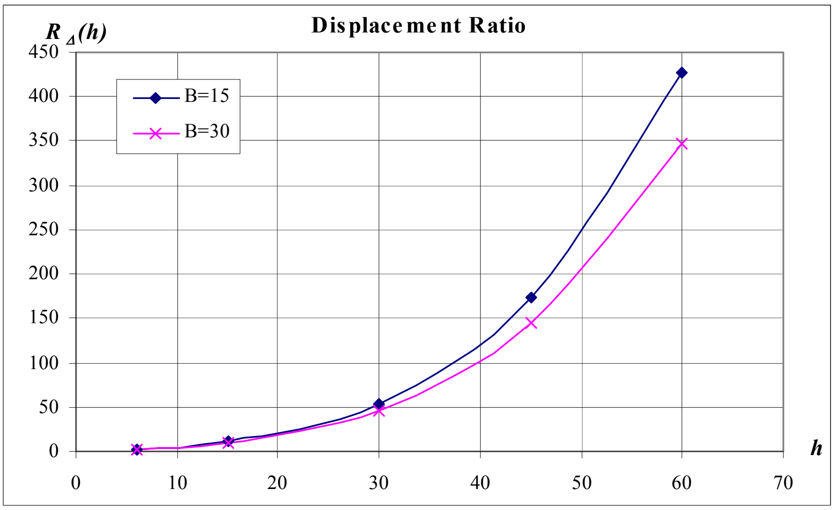

Fig 14. Case (4):Displacement Ratio

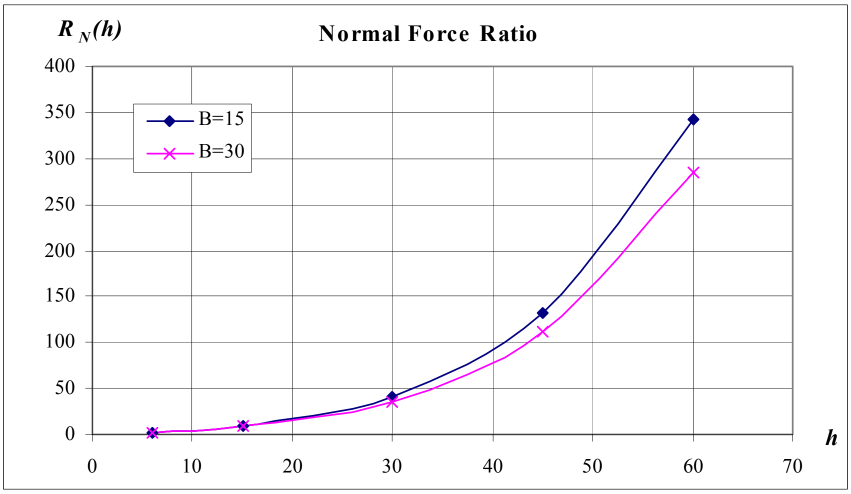

Fig 15. Case (4):Normal Force Ratio

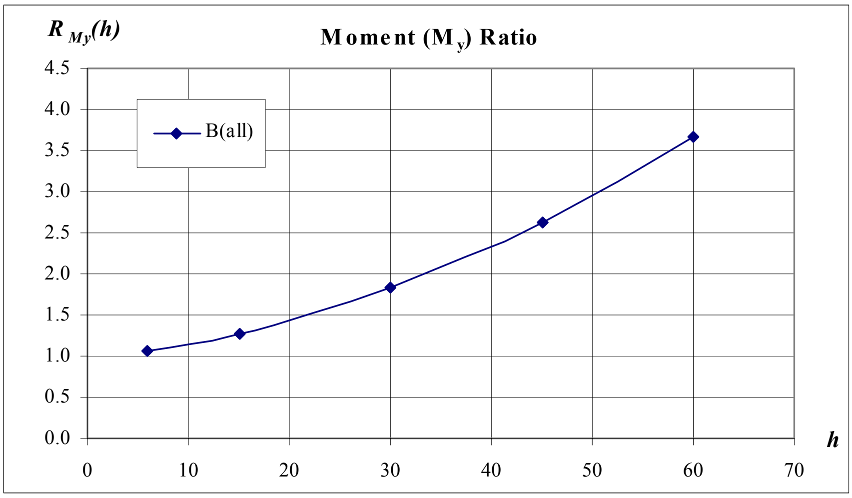

Fig 16. Case (4):Moment (My) Ratio

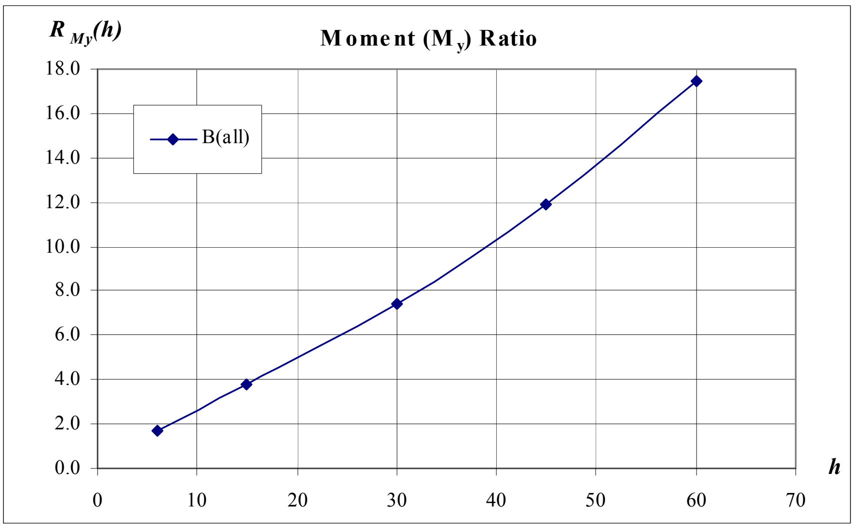

Fig 17. Case (4):Moment (My) Ratio - Shear Wall

Case (5): Symmetrical Dual Systems

Case 5 in Fig. 2 represents a plan layout of a dual system composed of two symmetrical shear walls and moment-resisting frames. Several heights are considered: *6, 15, 30, 45*, and *60*ms. For each height, widths of *30, 25, 20*, and *15*ms are investigated. The following relationships and figures govern the ratio of the full-scale building to the one-story model for the columns and shear wall:

(a) System Drift, Δ (Fig. 18)

all B, $f(h) = 0.0018h^3 - 0.0368h^2 + 0.9589h - 2.8051$ ………….. (16)

(b) Column Normal Forces (Fig. 19)

all B, $f(h) = 0.0021h^3 - 0.0756h^2 + 1.6297h - 5.7783$ ………..…… (17)

(c) Column Moments M_y (Fig. 20)

all B, $f(h) = 0.0005h^2 + 0.001h + 1.004$ ……………………...… (18)

(d) Shear Wall Normal Force (Fig. 21)

all B, $f(h) = 0.0031h^3 - 0.1131h^2 + 2.3748h - 8.9822$ ……..…. (19)

(e) Shear Wall Moment M_y (Fig. 22)

all B, $f(h) = 0.0026h^2 + 0.1451h + 0.8908$ ..…………………...… (20)

Figures 18-22 represent the ratios for the system drift Δ, column normal forces N, column moments M_y, shear wall normal forces N_{sh}, and shear wall moment M_y, respectively. Figure 4 shows the ratio for the shear force V_x in the columns and shear walls. All other straining actions in columns and shear wall are negligible.

CONCLUSIONS AND DISCUSSIONS

A simplified approach for approximate analysis of multi-story buildings against lateral loads is presented. A three-dimensional one story model that does not dispense any of the main features of the structural model is proposed. Using analysis charts and equations, it is proven possible to scale the drift and internal forces resulting from the proposed one story model to come up with the approximate corresponding values for a multi-story building up to 60 ms high. Different cases of eccentricities for buildings with and without shear walls are included. Different height-width ratios are incorporated. Several examples are used to validate that the method provides good approximations for the analysis of multi-story buildings.

The work paves the way for more research in this area in order to explore the effect of different parameters. Other cases of column-base supporting conditions and irregularities in stiffnesses or masses in the vertical direction need to be investigated. Other plan layouts and higher buildings may be studied. The issue of framing (beams, flat slabs, or hordi) and the issue of bracing (braced versus non-braced) need to be further investigated. Finally, the approach should be generalized to come up with internal forces and drift values for intermediate stories in addition to those of the lower story.

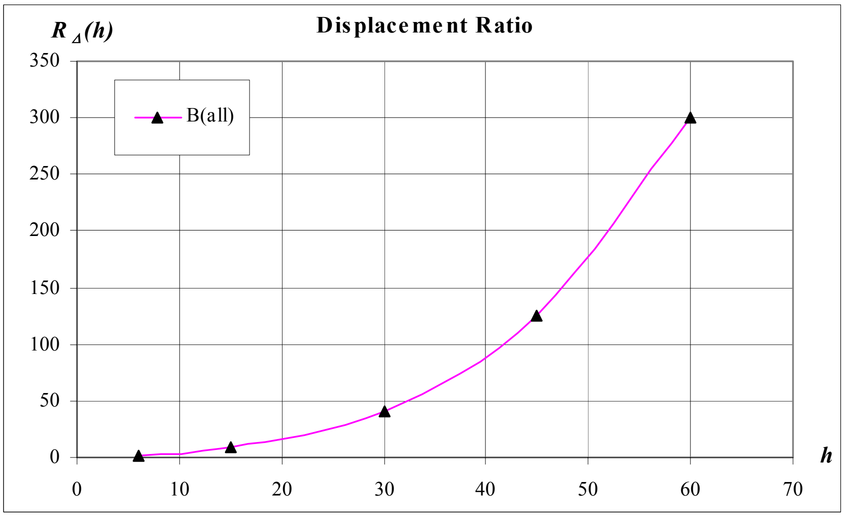

Fig 18. Case (5):Displacement Ratio

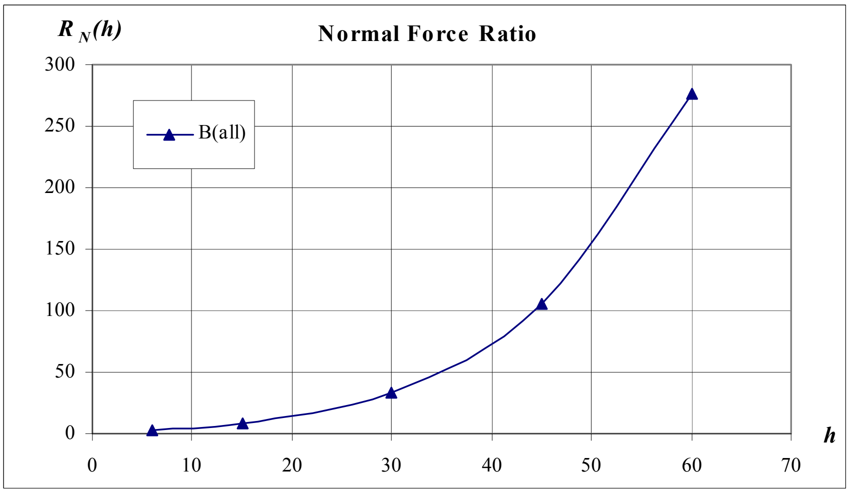

Fig 19. Case (5):Normal Force Ratio

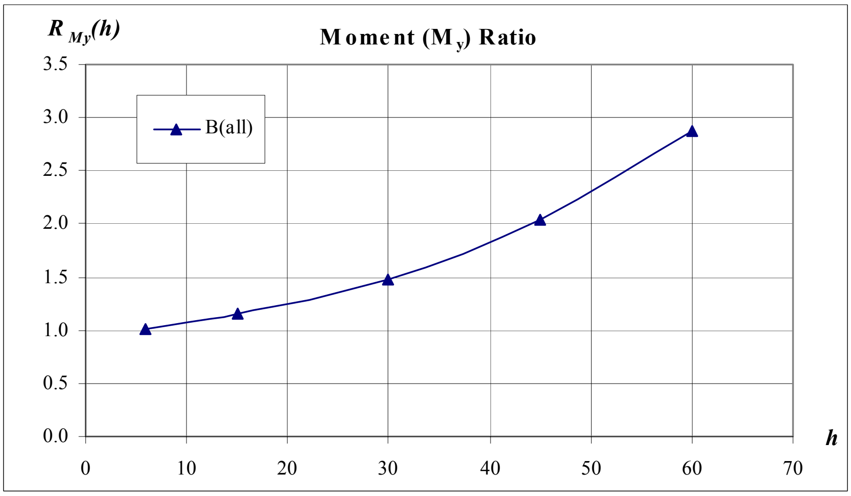

Fig 20. Case (5):Moment (My) Ratio

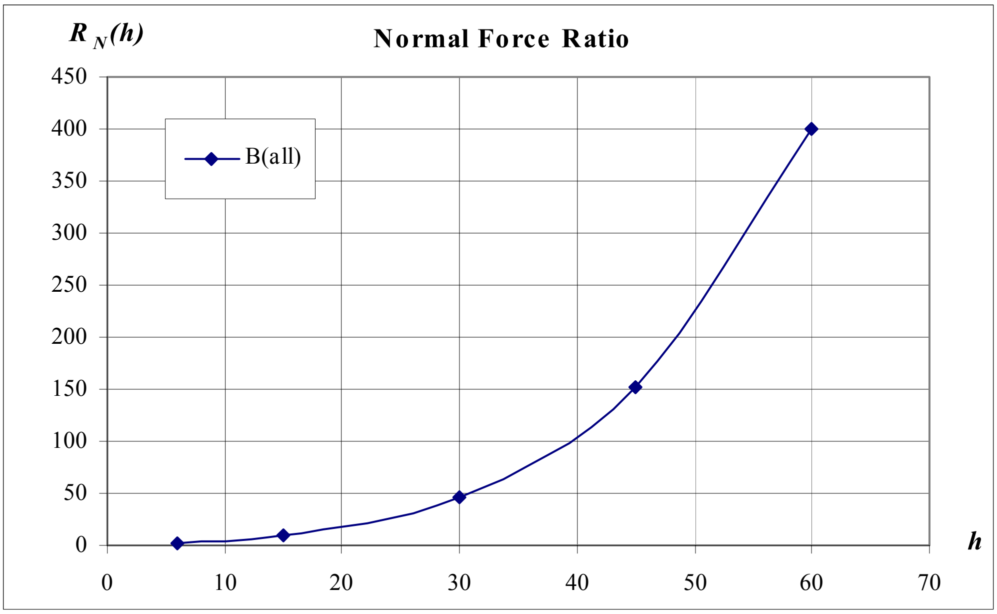

Fig 21. Case (5):Normal Force Ratio - Shear Wall

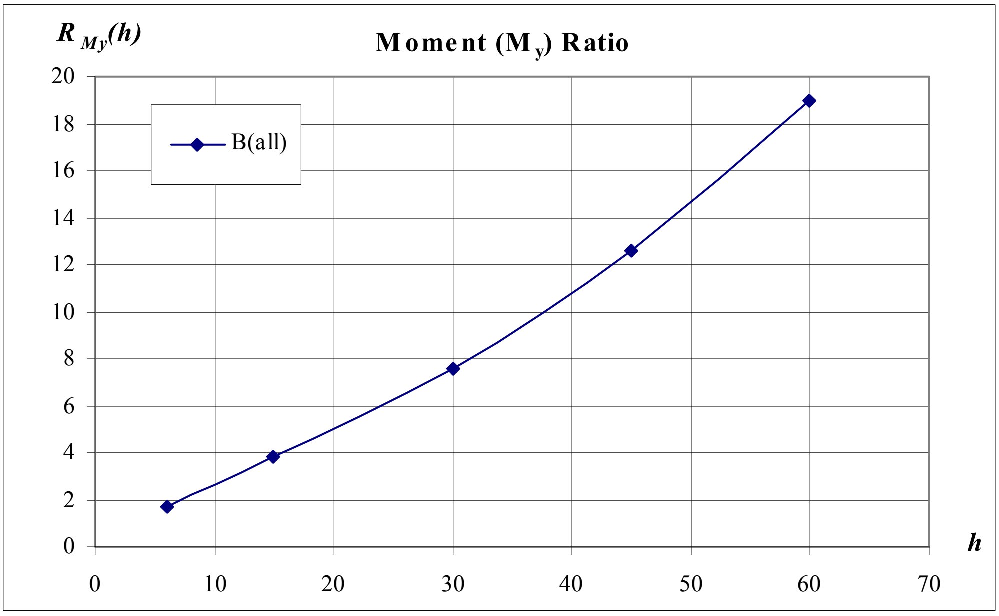

Fig 22. Case (5):Moment (My) Ratio - Shear Wall

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A SIMPLIFIED APPROACH FOR APPROXIMATE ANALYSIS AGAINST LATERAL LOADS

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ABSTRACT

A simplified approach that reduces the size of the problem to a more viable size is presented for estimating straining actions and drift values for preliminary design against lateral loads. The proposed technique does not dispense any of the main features that affect the structural behavior. The idea is an extension of the sub-frame method proposed by ACI for analysis against gravity loads. A one-story three-dimensional model of the actual building is proposed in order to provide estimates of forces, moments, and drift values for multi-story buildings. The results of a statistical study are used to develop analysis charts for framed systems with and without shear walls. Different cases of torsion eccentricities are considered. The study is conducted for up to 20-story buildings with different height-width ratios. Equations resulting from nonlinear regression analyses are presented in order to provide simplified expressions for preliminary analysis of multi-story buildings. Five cases are used to validate that the method provides good approximations that are adequate for design of multi-story buildings.

طريقة تقريبية لتحليل المنشآت المعرضة للأحمال الأفقية

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ملخـص

يعرض البحث طريقة تقريبية يتم التحكم بواسطتها في حجم المسألة الى حد بعيد وذلك لتقدير القوى الداخليـة والإزاحات الكلية بغرض التصميم المبدئي للمنشآت تحت تأثير الأحمال الأفقية. وتتميز الطريقة بأنها تأخذ في الإعتبار جميع العناصر الإنشائية الأساسية التي تؤثر على سلوك المنشأ دون إهمال أياً منها. وتعتبر الطريقـة إمتداداً للطريقة التقريبية التى يقدمها الكود الأمريكي لتحليل المنشآت تحت تأثير الأحمال الرأسـية. وتعتمـد الفكرة على حل النموذج الفراغي للدور الأرضي فقط بدلاً من حل النموذج الفرغي للمبنى كله ثم إستخدام قيم القوى والإزاحات لهذا النموذج في تقدير قيم القوى والإزاحات الكلية المناظرة في حالة حـل المبنـى كلـه. ويتوصل البحث –بناءاً على دراسة تحليلية- إلى مجموعة من المنحنيات ممكن إستخدامها في حالـة المبـاني المكونة من إطارات فقط أو من إطارات وحوائط قص معاً. ويعرض البحث أيضاً عدة حالات مـن التماثـل وعدم التماثل للنظام الإنشائي في المسقط الأفقي. وقد تمت دراسة مباني ذات إرتفاعات تصل إلى ٢٠ طابقـاً وذات نسب إرتفاع/عرض مختلفة. وتم إستنتاج مجموعة من المعادلات اللاخطية يمكن إستخدامها في التحليل التقريبي للمنشآت متعددة الطوابق. وقد استخدمت في الدراسة خمسة حالات مختلفة للتحقق مـن أن الطريقـة المقدمة تؤدى الى تقديرات جيدة للقوي الداخلية والأزاحات بحيث يمكن إستخدامها في تصميم المنشآت متعددة الطوابق.

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